

Distances in Agorea

A Short Story by:
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A wacky mind

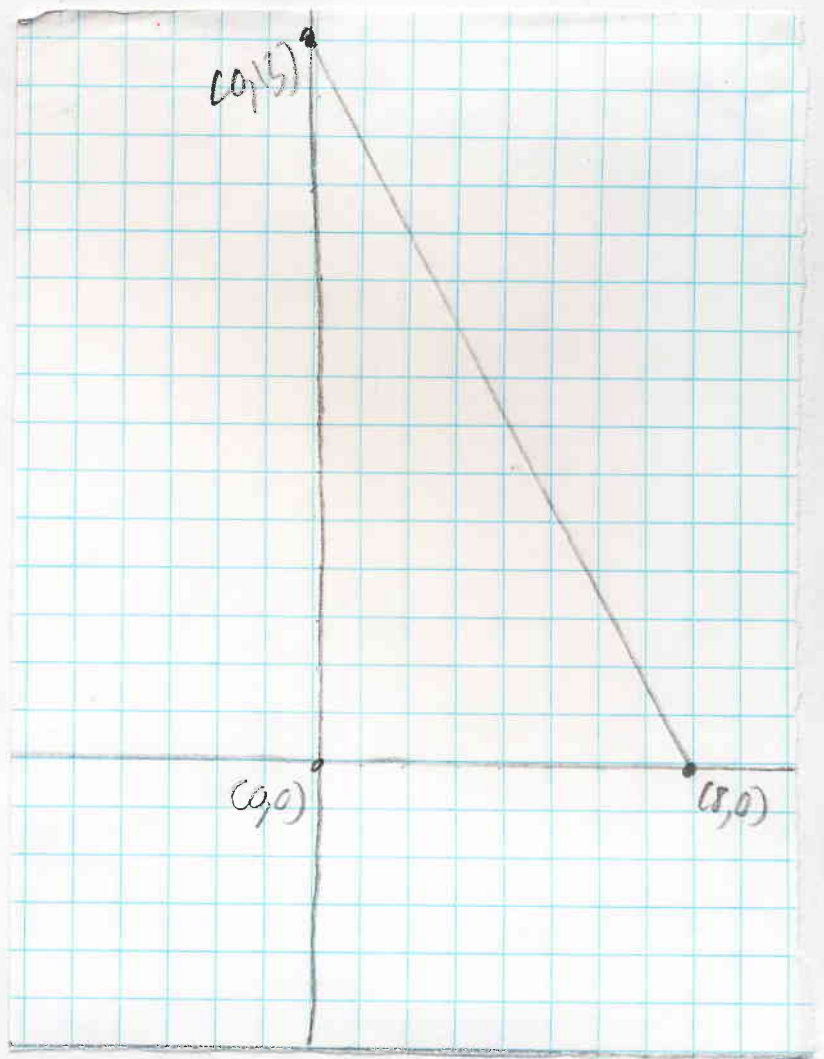
Illustrations by:
Patrick Deighan
Graph Paper
Rulers

Works Cited:
Geometry for Enjoyment and Challenge, McDougal Littell/
Houghton Mifflin. ©1991
The left side of my brain
Tufts of scratch paper and notes

Edited by:
Tom Deighan
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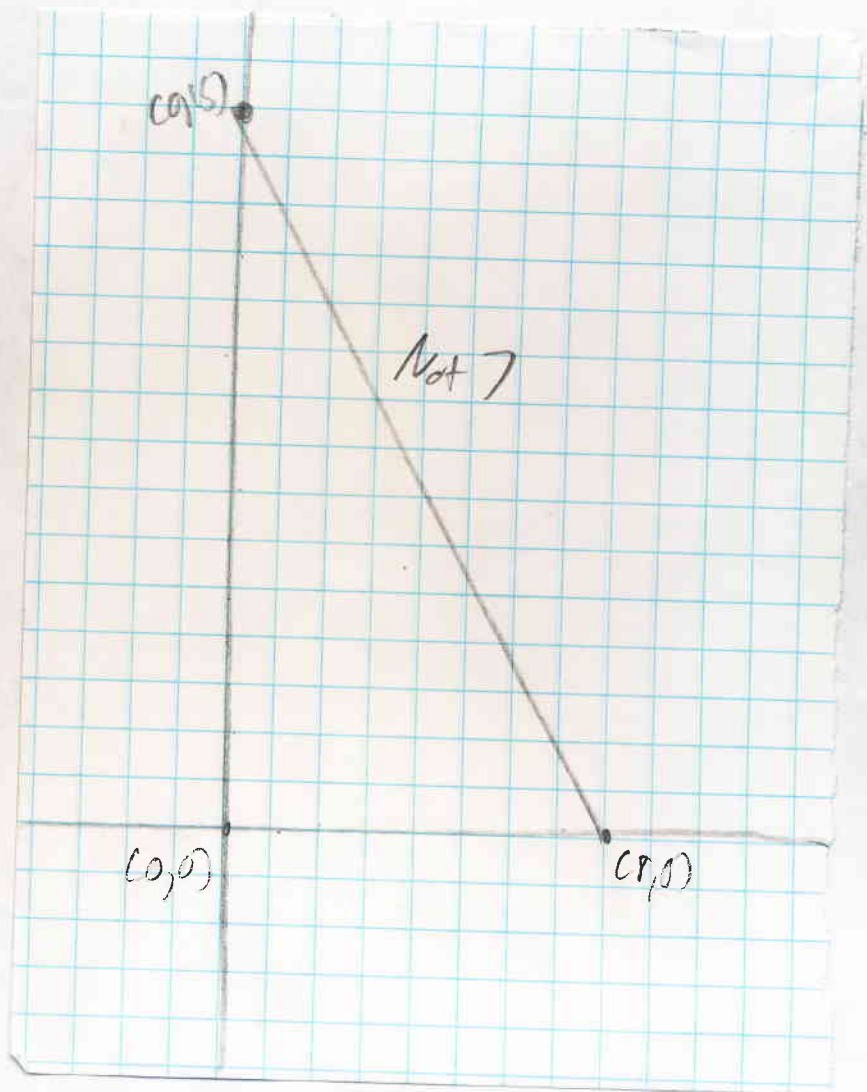
Publisher:
A Stapler

Paper made from 100% printer usage



There once was a land called Agorea. It was watched over by a scholarly young prince by the name of Pyth. Pyth of Agorea was especially fascinated by shapes, but he had a particular problem. "I know that walking straight from here to there must be quicker," he said, "yet the tiles cannot be measured direct that way, only horizontally and vertically. I wonder just how far it is." His good friend Dis, from the far-off land of Tance agreed with him. He told Dis that if ever a way were found to measure this length, it would be called distance, the shortest length between two points. The connected points also needed a name, so he called them vertices, because at the moment they were all vertical from where he stood. He called the vertex at the right angle the starting point and assigned it $(0,0)$. Since the first point was the point to his right he added 8 to the first number to give it $(8,0)$. He added fifteen to the other, making it $(0,15)$.

$$0 - 8 = -8$$
$$15 - 0 = 15$$
$$15 + -8 = 7$$



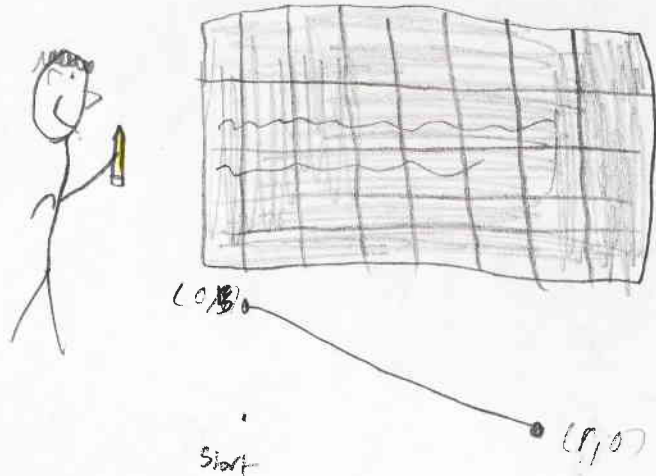
He figured that to find the distance he would need to find the differences between the two vertices away from him. He found that the difference of the two first and second numbers were -8, and 15. He thought maybe by adding them he could get the distance, however when he did so he got seven. "This line is not that long."

$$0-7 = -7$$

$$15-0 = 15$$

$$\frac{15}{-7}$$

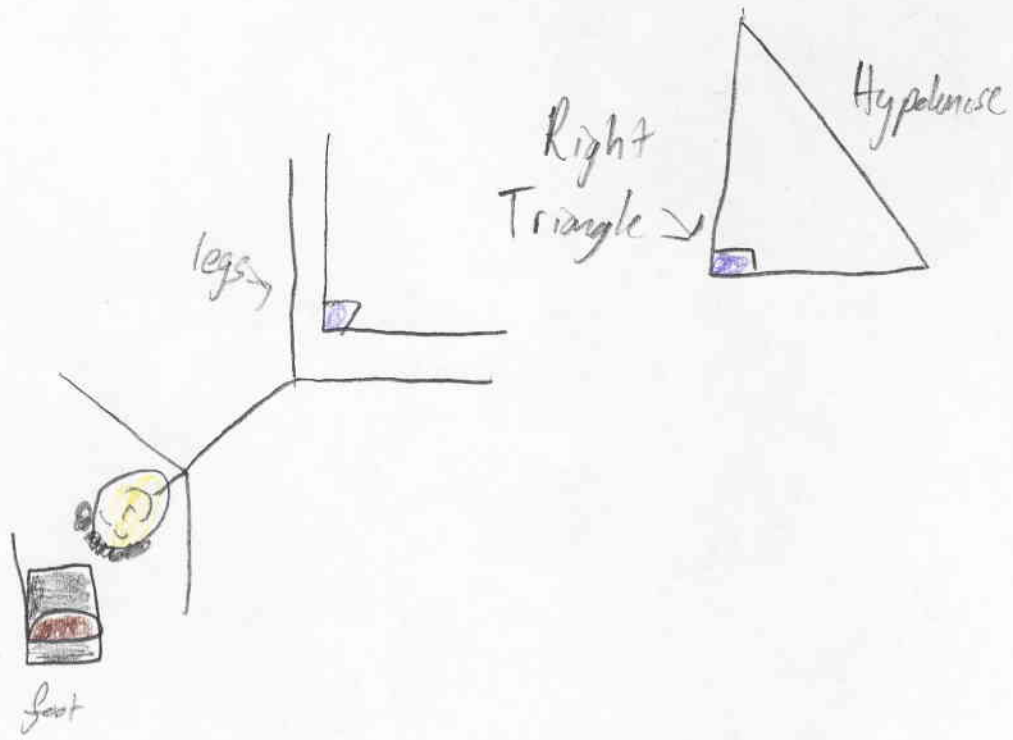
$$\sqrt{7} \approx 2.645751311$$



He remembered that square roots were always smaller than the number. So he attempted that with the number seven. He said "Yes, this seems about right" He declared this method the Distance formula, since it was used to find distance and wrote it upon the walls. "The difference of two horizontal points in an ordered pair plus the difference of the corresponding vertical points in the same pair taken by their square root. For example 0-8 plus 15-0 equals seven. Take the square root and find the distance below." For it was written above the area he had chosen to mark his experiment.



“Bravo!” cheered his people. But Hyp said to him, “Is that really the only way this can be done?” Pyth thought carefully and replied “Perhaps not. If one were to take the two lengths of the legs 15 and 8, rather than points, another way may be found.” “Show us!” cried the people. Pyth knew that the easiest way to measure the length would be to establish a polygon that included the length he wished to find. He remembered that a right angle was a 90 degree turn. Using this he would be allowed to have two sides that met at a ninety degree angle. In honor of his teacher, he called any two such lines perpendicular.



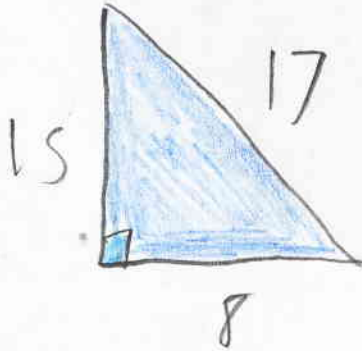
“Since each of these tiles is the length of my foot,” declared Pyth, “we shall recognize this length as a foot.” He enclosed the area with three sides, two of which were perpendicular and the third the length he wanted to know. The two perpendicular lines, he noted, were shorter than the third. Since they were about the size of his legs, he determined to call the two perpendicular lines in such a triangle legs. This triangle, he thought, should be called a right triangle, because of its one right angle. He thought that there should still be a name for the third side. He determined that in honor of his neighboring kingdom’s ruler Hyp, who encouraged him to explore all things beyond even the land of Tenuse, that it would be called Hypotenuse.

$$8 \cdot 8 = 64$$

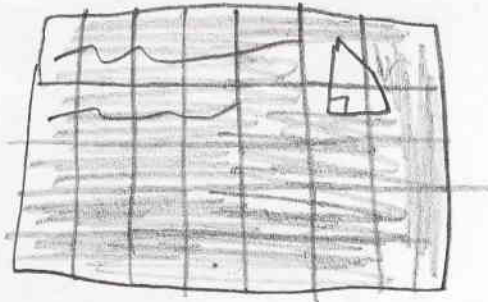
$$15 \cdot 15 = 225$$

$$\begin{array}{r} 225 \\ + 64 \\ \hline 289 \end{array}$$

$$\sqrt{289} = 17$$



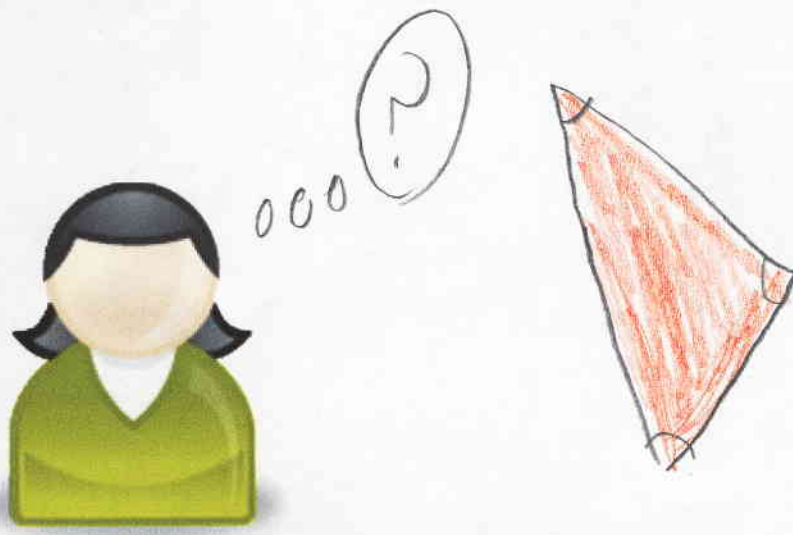
“This right triangle,” he said, “has two legs, of lengths 15 and 8 feet. It also has a hypotenuse, or a distance, which connects the other end of the legs.” With a definite shape laid out, Pyth was ready to begin his work. The hypotenuse appeared longer than either of the other two sides, so he determined to use some addition. He also thought that squares and square roots were very precise. Therefore the 8 and 15 were squared to get 64 and 225 respectively. When he added them he got 289. Upon finding its square root he found that the distance was seventeen. The reason for the different results, he determined, was that the line’s slope was changed to fit the triangle.



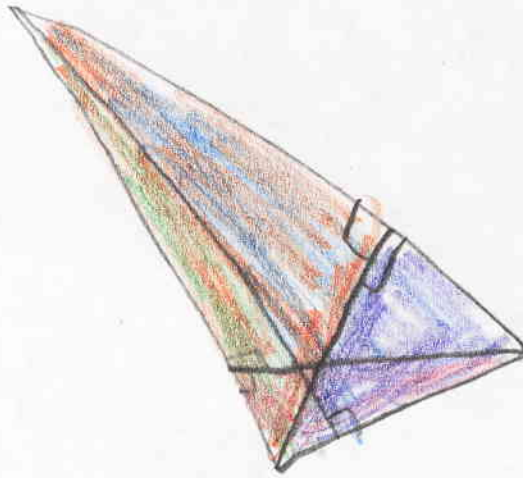
Pyth, Agorean!



The crowd went wild again and he wrote this upon the wall too. "Leg 1 squared plus leg 2 squared equals hypotenuse squared." This too must have a name. The crowd was chanting his name "Pyth, Agorean. Pyth, Agorean!" He thought at the crowds' chanting of his name and nationality. Pythagorean? Hmm. "From this point forward this method will be known as the Pythagorean Theorem!"

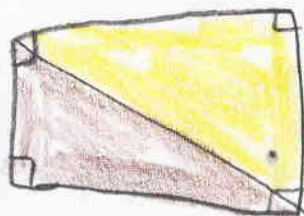


One person in the crowd seemed lost in thought so Pyth went forth to speak with her. “What troubles you, young one?” he inquired. “My name is Itu, of the foreign island of Des. We love your Theorem, but what if we wish to use it and there are no right angles? It wouldn’t work then.” After some computations Pyth discovered she was right. So he thought and found an idea.



Altitudes

“If one were to draw a line from a vertex on any triangle to the opposite side, they could create a right angle. See here.” He drew an irregular triangle in the ground and from a vertex drew a line perpendicular to the opposite side. “See now how we have two triangles to use the theorem in?” She looked and said, “Yes, but those lines need a name too.” He thought and said, “In honor of your thought we shall call a line drawn from a vertex perpendicular to the opposite side an altitude, alt meaning alternative.”



Her eyes brightened, “Thank you!” and she left to show her friends. He also showed how in quadrilaterals with right angles a diagonal from one vertex to the opposite could create two right triangles and the diagonal could then be measured. From that point forward, Pyth would be hailed as one of the greatest mathematical minds of all time.